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Report to the College Entrance Examination Board upon Elementary Algebra, Advanced Algebra, and Plane Trigonometry by the Commission on College Entrance Requirements in Mathematics:

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The Commission recommends that even though this Report be adopted by the Board at its meeting in November, 1922, examinations in conformity with the following requirements be held for the first time in June, 1924, and that it may be desirable perhaps for a year or two after that date to hold examinations also in conformity with the present requirements of the Board.

The Commission's Report on Plane Geometry and Solid Geometry is being printed separately.

ELEMENTARY ALGEBRA (PART I)

ALGEBRA TO QUADRATICS

1. The meaning, use, evaluation, and necessary transformations of simple formulas involving ideas with which the pupil is familiar, and the derivation of such formulas from rules expressed in words (1).*

The following are types of the formulas that may be considered:

$V=\frac{4}{3}\pi r^3$ (the sphere),

$A=\frac{1}{2}h(b+b')$ (the trapezoid),

$s=\frac{1}{2}gt^2$ (falling bodies),

$A=p(1+rt)$ (amount at simple interest),

$A=p(1+r)^t$ (amount at compound interest).

2. The graph, and graphical representation in general. The construction and interpretation of graphs (2).

The following are types of the material adapted to this purpose: statistical data; formulas involving two variables, such as

$$A=\pi r^2,$$

and

$$y=x^2+3x-2;$$

formulas involving three variables, but considered for the case in which an arbitrary value is assigned to one of them, as $V=\pi r^2h$ for a fixed value, say 4, of h.

3. Negative numbers; their meaning and use (3).

This requirement includes the fundamental operations with negative numbers and the interpretation of a negative result in a problem, provided such an interpretation is germane to the problem.

* The numbers refer to the Notes to Teachers.

4. Linear equations in one unknown quantity, and simultaneous linear equations involving two unknown quantities, with verification of results [4]. Problems [5].

The coefficients of a single linear equation in one unknown quantity may be literal fractions. In the case of simultaneous equations, literal coefficients are restricted to simple integral expressions, and to cases readily reducible to such expressions.

5. Ratio, as a case of simple fractions; proportion, as a case of an equation between two ratios; variation [6]. Problems.

6. The essentials of algebraic technique, including:

a) The four fundamental operations [7].

b) Factoring of the following types [8]:

(1) Monomial factors;

(2) The difference of two squares;

(3) Trinomials of the type x^2+px+q.

c) Fractions, including complex fractions of simple type [9].

The requirement includes complex fractions of about the following degree of difficulty:

$$\frac{p+\frac{a}{b}}{q-\frac{c}{d}}, \qquad \frac{\frac{a+3b}{c-5d}}{\frac{a-3b}{c+5d}}, \qquad \frac{\frac{a}{b}+\frac{c}{d}}{\frac{m}{n}-\frac{p}{q}}.$$

d) Numerical verification of the results secured under *a*), *b*), and *c*).

7. Exponents and radicals.

a) The proof of the laws for positive integral exponents.

b) The reduction of radicals, confined to transformations of the following types:

$$\sqrt{a^2b}=a\sqrt{b}, \qquad \sqrt{\frac{a}{b}}=\frac{\sqrt{ab}}{b}, \qquad \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}},$$

$$\frac{1}{\sqrt[3]{a}}=\frac{\sqrt[3]{a^2}}{a}, \qquad \frac{1}{\sqrt[3]{a^2}}=\frac{\sqrt[3]{a}}{a},$$

and to the evaluation of simple expressions involving the radical sign (10).

c) The meaning and use of fractional exponents, limited to the treatment of the radicals that occur under *b*) above.

d) A process for finding the square root of a number, but no process for finding the square root of a polynomial.

8. Numerical Trigonometry.

The use of the sine, cosine, and tangent in solving right triangles.

The use of four-place tables of natural trigonometric functions is assumed, but the teacher may find it useful to include some preliminary work with three-place tables.

It is important that the pupil should acquire facility in simple interpolation; in general, emphasis should be laid on carrying the computation to the limit of accuracy permitted by the table.

ELEMENTARY ALGEBRA (PART II)

QUADRATICS AND BEYOND

1. Numerical and literal quadratic equations in one unknown quantity [11]. Problems.

The requirement includes the solution of the general quadratic equation

$$ax^2+bx+c=0,$$

the conditions for the reality and for the distinctness of the roots, and the formulas for the sum and the product of the roots. Simple cases in which x is replaced by z^2 or by a binomial, and problems leading to quadratics, are also included; furthermore:

The interpretation of the graph of such an expression as x^2-3x+5, meaning thereby the graph of the corresponding equation,

$$y=x^2-3x+5.$$

2. The binomial theorem for positive integral exponents, with applications [12].

It is not intended under this topic to include problems involving irrational numbers, or surds, or the expansion of the powers of a binomial having more than one fractional coefficient. Such simple applications as that to compound interest are included.

3. Arithmetic and geometric series.

The requirement is limited to the formulas for the nth term, the sum of the first n terms, the value of such an infinite decreasing geometric series as $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$, and to simple applications [13].

4. Simultaneous linear equations in three unknown quantities.

The coefficients may be integers, numerical fractions, or algebraic monomials.

5. Simultaneous equations, consisting of one quadratic and one linear equation, or of two quadratic equations of the following types:

$$\begin{cases} ax^2+by^2=c, \\ xy=h; \end{cases} \qquad \begin{cases} a_1x^2+b_1y^2=c_1, \\ a_2x^2+b_2y^2=c_2. \end{cases}$$

These may be expressed in other forms, such as

$$x^2=(r+y)\ (r-y), \qquad xy=r^2.$$

The coefficients may be integers, numerical fractions, or algebraic monomials.

Graphical treatment is expected in the cases of equations of the types

$$x^2+y^2=a^2, \qquad x^2-y^2=a^2, \qquad xy=a, \qquad y^2=ax.$$

6. Exponents and radicals.

a) The theory and use of fractional, negative, and zero exponents (14).

b) The rationalization of the denominator in such fractions as

$$\frac{a+\sqrt{b}}{a-\sqrt{b}}.$$

c) The solution of such equations as

$$x+\sqrt{x-5}=11.$$

7. Logarithms (15).

a) The fundamental formulas;

b) Computation by four-place tables;

c) Applications to the trigonometry of the right triangle.

ADVANCED ALGEBRA

a) THEORY OF EQUATIONS

The theorem that an equation of the nth degree has n roots, if every such equation has one root. Factoring of polynomials in one variable, and the remainder theorem. The coefficients as symmetric functions of the roots. Simple transformations of equations, limited to the removal of the second term, and increase of the roots by a given number and multiplication of the roots by a given factor. Conjugate complex roots of equations with real coefficients.

Equations with whole numbers or fractions as coefficients. Condition for a rational root [16].

Approximate solution of numerical equations. Descartes's Rule of Signs. Preliminary location of the roots by the graph. Determination of the roots to two or three significant figures [17].

b) DETERMINANTS

Definition of determinants of the second and third orders by the explicit polynomial formulas [18]. Evaluation of such determinants by the familiar rules [19]. The simple transformations[20], proved directly from the definition, and illustrated by examples in which the elements of the determinant are small integers.

Determinants of the fourth order; their evaluation and transformations [21].

Application to linear equations: (*i*) non-homogeneous equations in two, three, and four unknowns; (*ii*) homogeneous equations in two and three unknowns; the treatment to include all cases in which the number of equations does not exceed the number of unknowns [22]. The case of compatibility of three non-homogeneous equations in two unknowns is also included [23].

c) BRIEF TOPICS

Complex numbers, numerical and geometric treatment [24].

Simultaneous Quadratics [25].

Scales of Notation [26].

Mathematical Induction [27].

Permutations and Combinations. Probability [28].

TRIGONOMETRY

1. Definition of the six trigonometric functions of angles of any magnitude, as ratios. The computation of five of these ratios from any given one. Functions of 0°, 30°, 45°, 60°, 90°, and of angles differing from these by multiples of 90° (29).

2. Determination, by means of a diagram, of such functions as $\sin(A+90°)$ in terms of the trigonometric functions of A.

3. Circular measure of angles; length of an arc in terms of the central angle in radians.

4. Proofs of the following fundamental formulas (30), and of simple identities derived from them.

a) The Ratio Formulas:

$$\tan x=\frac{\sin x}{\cos x},\qquad \cot x=\frac{\cos x}{\sin x},\qquad \sec x=\frac{1}{\cos x},\qquad \csc x=\frac{1}{\sin x},$$

$$\cot x=\frac{1}{\tan x};$$

b) The Pythagorean Formulas:

$$\sin^2 x+\cos^2 x=1,\qquad 1+\tan^2 x=\sec^2 x,\qquad 1+\cot^2 x=\csc^2 x;$$

c) The Addition Theorems:

$$\sin(x+y)=\sin x\cos y+\cos x\sin y,$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y,$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y};$$

d) The Double-Angle Formulas, for $\sin 2x$, $\cos 2x$, $\tan 2x$.

5. Solution of simple trigonometric equations of the general order of difficulty of the following (31):

$$6\sin x+\cos x=2;\qquad \cos 2x=\sin x;\qquad \tan(x+30°)=\cot x.$$

6. Theory and use of logarithms, without the introduction of work involving infinite series. Use of trigonometric tables, with interpolation[32].

7. Derivation of the Law of Sines and the Law of Cosines.

8. Solution of right and oblique triangles (both with and without[33] logarithms) with special reference to the applications. Value will be attached to the systematic arrangement of the work.

NOTES TO TEACHERS

It is the purpose of the College Entrance Examination Board to set its requirements in Mathematics in such a way that the work of preparation for them shall be, at the same time, of the greatest intrinsic value to the pupil in connection with his subsequent work in mathematics, physics and engineering, science, economics, and other subjects which make use of mathematics. It appears that this purpose will best be served by specifying the minimum requirement in each subject and leaving to the teacher free hand for developing in the pupil power, and mastery of the subject. To prepare pupils satisfactorily for the examinations, teachers will, therefore, find it necessary to extend the work somewhat beyond the limits set in each topic.

It is not practicable to specify in advance all the non-mathematical terms that may be used in problems on the applications of mathematics, and it will never be possible to escape altogether the difficulties arising from the fact that words and expressions which form a part of the language of common life for one group of students may be strange to another group; but it seems worth while to specify that in connection with problems in physics the candidate in Trigonometry will be expected to know the principle of the parallelogram of forces, and for problems involving the points of the compass, although he will not be called upon to interpret such expressions as "northeast by east a quarter east," he will be expected to know the names of the eight principal directions—north, northeast, east, and so on to northwest.

The order in which the topics are listed is not intended to imply any recommendation as to the order of presentation in teaching. For example, the teacher may prefer, in algebra, to begin with work in oral algebra and with problems leading to simple equations.

The formal statement of the requirements is supplemented by the following running commentary.

ELEMENTARY ALGEBRA

In conformity with the general principles above laid down, a less extensive treatment of certain purely algebraic topics than has hitherto been the practice in Elementary Algebra is herewith specified, in order that time may be saved for the introduction of intuitive geometry and numerical trigonometry, as well as for better emphasis on the great basal principles of algebra. The examinations are expected to be more searching with respect to the parts of algebra that will be used by the pupil in his later work, and particularly with respect to his ability to solve applied problems, to handle formulas, to interpret graphs, and to solve the types of equations which he will subsequently need to use.

The amount of time and the degree of maturity needed in preparing for these examinations are expected to correspond to the present requirements.

The requirements in algebra here formulated represent in each case the minimum requirement in that topic. Just as the power of the candidate in geometry is tested by originals not included in any standard list of propositions, so in algebra questions on any topic, which go somewhat beyond the stated requirement, but which are within the power of a candidate properly trained to meet that requirement, may form a part of the examination.

ELEMENTARY ALGEBRA (PART I)

ALGEBRA TO QUADRATICS

1. In the work done with formulas, the general idea of the dependence of one variable upon another should be repeatedly emphasized. The illustrations should include formulas from science, mensuration, and the affairs of everyday life. Throughout the course, there should be opportunity for a reasonable amount of numerical work and for the clarification of arithmetical processes.

2. The pupil should be required to construct the graph of such an expression as

$$x^2-2x+3,$$

i.e., to plot the curve

$$y=x^2-2x+3,$$

and to interpret the meaning of any graph constructed.

3. The relation of real numbers to points on a line should be made clear, the cardinal principle being that to every such number corresponds a point on the scale, and conversely. In this way, the relation of positive to negative numbers, of integers to fractional and irrational numbers, and of approximations to the values of both rational and irrational numbers will be rendered intelligible.

4. Beside numerical linear equations in one unknown, involving numerical or algebraic fractions, the pupil will be expected to solve such literal equations as contribute to an understanding of the elementary theory of algebra. For example, he should be able to solve the equation

$$s=\frac{ar^n-a}{r-1} \qquad \text{for } a.$$

In the case of simultaneous linear equations, he should be able to solve such a set of equations as

$$ax+by=k,$$
$$cx+dy=l,$$

in order to establish general formulas. But the instruction should include a somewhat wider range of cases, as for example:

$$\begin{cases} ax+(a+b)y=ab, \\ ax+(a-b)y=-ab; \end{cases} \quad \text{or} \quad \begin{cases} ax+by=ab, \\ x+\left(1+\dfrac{b}{a}\right)y=a. \end{cases}$$

The work in equations will include cases of fractional equations of reasonable difficulty; but, in general, cases will be excluded in which long and unusual denominators appear and in which the highest common factor of the denominators, or the lowest common denominator, cannot be found by inspection.

5. Problems in linear equations, as in ratio, proportion, and variation, will whenever practicable be so framed as to express conditions that the pupil will meet in his later studies. Nevertheless, such studies are generally so technical as to render it impossible to use the phraseology and laws that represent real situations, particularly in connection with proportion and variation. On this account it must be expected that the problems will frequently involve situations that are manifestly fictitious; but even such problems will nevertheless serve the purpose of showing the pupil how to translate from algebraic symbols to ordinary written language, and vice versa, and to use algebraic forms to aid in the solution of problems.

6. It is not considered necessary to treat ratio and proportion as a distinct topic. The pupil is expected to look upon

$$\frac{a}{b}=\frac{c}{d}$$

as a fractional equation, or as a formula involving fractions, and to transform it accordingly. Similarly, he is expected to look upon the equation $y=cx$ as representing simple variation without rendering it obscure by the use of such a form as $y \propto x$.

Such terms as "alternation," "composition," etc., should not be introduced. The corresponding theorems, the substance of which can be presented without the use of the technical terms, are of value for some applications in geometry and other branches, but are not included in the requirements in algebra.

7. It is not expected that pupils will be called upon to perform long and elaborate multiplications or divisions of polynomials, but that they will have complete mastery of those types that are essential in the subsequent work with ordinary fractional equations, and with such other topics as are found in elementary algebra. In other words, these operations should be looked upon chiefly as a means to an end.

8. Factoring has been reduced to the cases needed in later work. Under (3), only simple cases, like

$$x^2-5x+6 \qquad \text{and} \qquad x^2+x-2$$

are contemplated. A problem may, however, combine two or more of the cases here suggested, as in the polynomial

$$x^4+4x^3-y^2+4x^2, \qquad \text{or} \qquad 2ax^2-4ax+2a,$$

or

$$x^4+4x^3-x^2y^2+4x^2.$$

The teacher may wish to use such examples as

$$a^2-b^2+a-b \qquad \text{or} \qquad xy+2x+3y+6$$

for purposes of instruction; but these cases are not included in the requirement.

9. Complex fractions may often be treated advantageously as cases in the division of simple fractions, but usually it is more convenient, in practice, to multiply both numerator and denominator of the complex fraction by a factor that will reduce it to a simple fraction.

The meaning of the operations with fractions should be made clear by numerical illustrations, and the results of algebraic calculations should be frequently checked by

numerical substitution as a means to the attainment of accuracy in arithmetical work with fractions.

10. In all work involving radicals, such theorems as $\sqrt{ab}=\sqrt{a}\sqrt{b}$ and $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ may be assumed. Proofs of these theorems should be given only in so far as they make clearer the reasonableness of the theorems; and the reproduction of such proofs is explicitly excepted from the requirements here formulated.

ELEMENTARY ALGEBRA (PART II)

QUADRATICS AND BEYOND

11. The coefficients of the equation may be common fractions or simple literal fractions. In such cases it is usually best to begin by freeing the equation of fractions. The method of solution by completing the square should be the one primarily used, with emphasis on the systematic arrangement of the necessary steps and computations. Equations of the form

$$ax^2+bx=0$$

should be solved by factoring; beyond this, the method of solution by factoring should be restricted to simple cases, such as

$$x^2-5x+6=0, \qquad \text{and} \qquad x^2+x-2=0.$$

It should be understood that the recommendations here made do not exclude the method of solution by the formula.

12. The pupil should be able to write the first few terms of the expansion of $(a+b)^n$ for any positive integral value of n, although in general it will be sufficient to take $n \leqq 8$.

13. In connection with the infinite decreasing geometric series, such illustrations as that of the repeating decimal should be used, but the subject of repeating decimals will not be required on the examinations. One important reason for the inclusion of this topic is that it serves as an introduction to the idea of the limit.

At some point in the course, the expression of a^n-b^n as the product of $a-b$ and a second factor should be taken up, and this may be done in connection with the formula for the sum of n terms of a geometric progression. It is sufficient to restrict n to values not exceeding 8. The corresponding factoring of a^n+b^n when n is odd should be included.

14. Fractional, negative, and zero exponents should be presented as cases in which a symbol is defined in such a way as to give universality to a law already established for certain special cases. That is, because

$$a^m a^n = a^{m+n}; \qquad (a^m)^n = a^{mn}; \qquad a^m b^m = (ab)^m,$$

when m and n are positive integers,

$$a^{\frac{p}{q}}, \qquad a^{-m}, \qquad a^0$$

are so defined as to render these laws permanent.

15. The graph of $y = 10^x$ may profitably be used for the purpose of explaining the theory. Only the base 10 need be considered in presenting the subject, but other bases may be used for purpose of illustration.

Proofs of the theorems

$$\log P + \log Q = \log PQ;$$
$$\log P^n = n \log P;$$

may be required on examinations; but proofs of the derived theorems, such as

$$\log \frac{P}{Q} = \log P - \log Q, \qquad \text{and} \qquad \log \sqrt[n]{P} = \frac{1}{n} \log P,$$

although these can readily be deduced from the first two, will not be required in examinations.

The requirement includes the computation of such an expression as

$$\sqrt[3]{\frac{1.967 \times 0.9834}{3.142}};$$

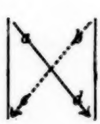
the calculation of s when the formula $v^2 = 2gs$ is given, and values of v and g are given to three or four significant figures; and the complete solution of a right triangle in which the given parts have been measured to four significant figures. It excludes the solution of exponential equations, like $2^x = 3$, and the computation of such expressions as $6.821^{0.1489}$.

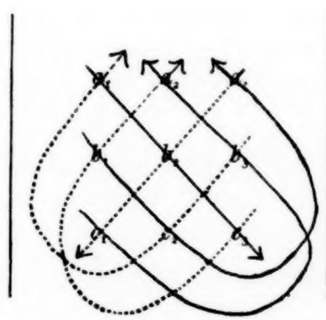
16. The requirement relating to a rational root is restricted to the case in which the coefficients are integers and the coefficient of the highest power of the unknown is unity.

17. After a root has been shown to lie between two values, x_1 and x_2, which are fairly near together, the points of the graph whose x-coördinates are x_1 and x_2 are computed, and the chord determined by these points is drawn. The intersection of this chord with the axis of x yields the next approximation. A graphical solution of the problem may often be used with profit. For closer approximations to the root, this or other methods may be used. But the elaborate developments suggested by the name of "Horner's Method," which involve multiplying the roots by powers of 10, constructing a special array for the numerical work (over and above the scheme for reducing the roots by a given number), and finally "contracting" the work are not a part of the requirement.

18. That is, $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, and the six-term formula for determinants of the third order.

19. Namely,





20. Such as adding columns, etc.

21. It is important that the pupil should know that a determinant of the fourth order cannot be evaluated by such a scheme as that mentioned under Note 19, but is evaluated by expansion according to the elements of a row or a column. On the other hand, by transformations such as those of Note 20, some elements can be reduced to 0 and the numerical work thus abbreviated.

22. Thus, under (*i*), the pupil will be expected to know in what the solution of the equations

$$a_1x+b_1y+c_1=0,$$
$$a_2x+b_2y+c_2=0,$$

consists, both when the determinant of the coefficients,

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix},$$

has a value different from 0 and when it equals 0.

In order to avoid too great detail, the requirement is restricted to the case, when the determinant of the equations vanishes, in which at least one (first) minor is different from 0. But the instruction should make clear to the pupil how he can proceed in any numerical case and find the complete solution. Thus, under (*ii*), the solution of the equations

$$3x+4y-2z=0,$$
$$2x+3y+4z=0,$$
$$5x+7y+2z=0,$$

consists in giving to z an arbitrary value and then determining x and y in terms of z from the first two equations.

23. It is desirable that the extension of all the foregoing definitions and theorems to more general cases be pointed out; but all such extensions, as also the rule for the multiplication of determinants, are explicitly excluded from the requirement.

24. The reduction of the sum, difference, product, and quotient of two complex numbers to the form $a+bi$. The geometrical construction of the sum and difference, but not of the product and quotient, of two complex numbers.

25. Two equations of the type

$$a_1x^2+b_1xy+c_1y^2=d_1\,,$$
$$a_2x^2+b_2xy+c_2y^2=d_2\,.$$

26. The point of this requirement is the fact that the base 10 is accidental, that any other base greater than 1 could be used, and that computation would be performed by the aid of the addition table and the multiplication table, as is customary with the base 10.

27. A clear exposition of this important method of reasoning. Applications to such problems as summing the series

$$1^2+2^2+3^2+\ .\ .\ .\ +n^2,$$

and

$$1^3+2^3+3^3+\ .\ .\ .\ +n^3;$$

the derivation of the formula for

$$(a_1+a_2+\ .\ .\ .\ +a_n)^2;$$

and, possibly, the proof of the binomial theorem for a positive integral exponent; the simpler applications are the more illuminating, and the application to the proof of the binomial theorem will not be required in examinations.

28. Permutations and combinations, restricted to the case of n objects, all of which are different. Probability, restricted to problems of moderate difficulty.

The time should be divided about equally between the three parts, *a*), *b*), *c*), with possibly slightly less than one-third the time given to *c*). The Brief Topics should really, as the name implies, be treated briefly in the course, and their

importance should not be overrated on the question paper by the setting of difficult questions. Their value lies in their mathematical content, rather than in their being a means of further developing technique.

It is not contemplated that the total requirement should be greater than that in Trigonometry or Solid Geometry.

TRIGONOMETRY

29. The values of these functions should be read from diagrams, and not made merely a matter of memorizing.

30. This is a minimum list to be memorized. With respect to formulas *c*), the requirement is restricted to the case that x and y are both acute; but the instruction should include the generalization of the formulas to the case of any angles by the addition of $90°$.

In restricting explicitly the formulas to be memorized, the object has been to give the pupil perspective and to aid him in an efficient study of the subject. The Board does not wish to discourage the pupil from memorizing certain other formulas when it is convenient for him to do so. But these formulas should hold distinctly a secondary place in the pupil's mind, and they should not be allowed to compete with the fundamental formulas.

31. The instruction should include an explanation of the notation $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\cot^{-1} x$ on the basis of the definition:

$$y = \sin^{-1} x, \quad \text{if} \quad x = \sin y,$$

etc., y being measured in terms of radians, and the further definition of the *principal value* of each of these functions should be pointed out. But this topic is not included in the requirement.

32. The use of four-place logarithmic and trigonometric tables is recommended, but both four-place and five-place tables will be furnished the candidates in the examinations of the Board.

The computation of such expressions as $6.821^{0.1489}$, and the solution of exponential equations, like $2^x = 3$, are included in this requirement.

33. This implies the case in which the sides of the triangle are represented by small integers, or in which the data respecting the sides are correct only to two significant figures. This case would naturally be considered at the beginning, before logarithms are introduced, for it contributes to an appreciation of the actual geometrical magnitudes and the relations between them.

Graphical solutions by means of scale and protractor should be freely used as checks.

Both the division of the angle into minutes and seconds and the decimal subdivision should be taught, but only one is required. Tables corresponding to each division will be furnished at the examination of the Board.

The use of slide rules in the instruction is to be encouraged, but they will not be permitted in the examination for the reason that, when some candidates use slide rules and some do not, it is impossible to mark the papers justly.

FORMULAS OF TRIGONOMETRY

1. The Ratio Formulas:

$$\tan x=\frac{\sin x}{\cos x}, \qquad \cot x=\frac{\cos x}{\sin x}, \qquad \sec x=\frac{1}{\cos x}, \qquad \csc x=\frac{1}{\sin x},$$

$$\cot x=\frac{1}{\tan x}.$$

2. Formulas read off from a figure; e.g., $\cos(90^\circ+x)$, $\sin(-x)$, etc., in terms of the trigonometric functions of x.

3. The Pythagorean Formulas:

$$\sin^2 x+\cos^2 x=1,$$
$$1+\tan^2 x=\sec^2 x,$$
$$1+\cot^2 x=\csc^2 x.$$

4. The Addition Theorems:

$$\sin(x+y)=\sin x\cos y+\cos x\sin y,$$
$$\cos(x+y)=\cos x\cos y-\sin x\sin y,$$
$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}.$$

5. The Double-Angle Formulas:

$$\sin 2x=2\sin x\cos x,$$
$$\cos 2x=\cos^2 x-\sin^2 x,$$
$$=2\cos^2 x-1,$$
$$=1-2\sin^2 x,$$
$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}.$$

6. The Half-Angle Formulas, *a*) in Implicit Form:

$$1-\cos x=2\sin^2\frac{x}{2},$$

$$1+\cos x=2\cos^2\frac{x}{2},$$

$$\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}.$$

b) In Explicit Form:

$$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}},$$

$$\cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}},$$

$$\tan\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}.$$

7. *a*) Law of Sines:

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

b) Law of Cosines:

$$a^2=b^2+c^2-2bc\cos A.$$

8. Sums and Products.

$$\sin x+\sin y=2\sin\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y),$$
$$\sin x-\sin y=2\sin\tfrac{1}{2}(x-y)\cos\tfrac{1}{2}(x+y),$$
$$\cos x+\cos y=2\cos\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y),$$
$$\cos x-\cos y=-2\sin\tfrac{1}{2}(x+y)\sin\tfrac{1}{2}(x-y).$$

9. Special Formulas for Triangles:

a) Law of Tangents:

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}.$$

b) For the case in which the three sides are given:

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}},$$

$$\text{Area}=\sqrt{s(s-a)(s-b)(s-c)},$$

$$s=\tfrac{1}{2}(a+b+c).$$

Formulas 1, 3, 4, and 5 (and also 7) should be memorized, like the multiplication table. The proofs of 1 and 3 are immediate. In the case of the addition theorems, the proof for the sine and the cosine of the sum of two acute angles should be given and the formulas extended to the case of any angles by adding 90° to x or y. As already noted, the proof will be required on examinations only for the case in which x and y are both acute. The third formula, 4, and the formulas, 5, follow at once from these. The formulas for $\sin(x-y)$, etc., are obtained by replacing y by $-y$ and making the obvious reductions. These formulas need not be memorized, but should be deduced when required.

Formulas 2 include $\sin(x\pm 90^\circ)$, $\sin(90^\circ-x)$, $\sin(x+180^\circ)$, $\sin(180^\circ-x)$, $\sin(-x)$, etc., and the corresponding cases for the cosine, tangent, and cotangent. None of the formulas should be memorized, but each should be read off from the appropriate figure, which the pupil soon comes to visualize without pencil and paper.

The half-angle formulas, 6, should be deduced when needed from the double-angle formulas, 5. The double signs in 6 *b*) depend on the magnitude of the angle and cannot be grouped automatically.

Formulas 7 should be memorized, as already noted, and the pupil should be familiar with the proofs.

Formulas 8 are not to be memorized, although they are occasionally used in practice, and it is well for the pupil to know that such formulas exist. Their proof affords a useful exercise in the manipulation of the trigonometric identities.

Any triangle can be solved by the law of sines and the law of cosines, and these formulas are to be preferred when the sides are represented by simple numbers. Formulas 9 are well adapted to logarithmic computation in the cases to which they apply. The pupil should be able to use them, or formulas equivalent to them; but he is not required to memorize them, or to learn their proof. The formula for the area of a triangle when two sides and the included angle are given, namely,

$$\text{Area} = \tfrac{1}{2}ac \sin B,$$

is not included, since it should be read off, whenever needed, immediately from the figure.

PROVISIONAL STATEMENT

GEOMETRY

Beside the present requirements in Plane Geometry and Solid Geometry the Commission expects to report two new requirements, one, in a combined course in Plane and Solid Geometry, the other, an alternative course in Solid Geometry.

In this latter course, formal demonstrations will be required only for a restricted list of propositions, selected for special reasons. On the other hand, greater stress will be laid on the geometrical information of the candidate, his ability to visualize space figures and to represent them on paper, and his facility in solving problems of mensuration.

Committees of the Commission are at work on syllabi, which will make precise the requirement in book work and in general knowledge of the subject-matter.

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